COST EFFECTIVENESS IN SILVER VERSUS GOLD RESTORATIONS OF CUSP FAILURES

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Monterey, California

THESIS

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by

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Cost Effectiveness in Silver Versus
Gold Restorations of Cusp Failures

by

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ABSTRACT

A model of cusp breakdown in the eight functional molars and eight bicuspids of career officers in military service was developed on the basis of empirical data obtained from complete dental histories of career personnel. Using this model and cost estimates of materials and labor, it was determined that the expected cost of silver restorations will exceed the expected cost of a gold restoration at a minimum service life of twenty years with the expected cost of silver restorations becoming increasingly larger than a gold restoration as service life increases.

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I. INTRODUCTION AND BACKGROUND

A. GENERAL STATEMENT OF THE PROBLEM

The cost effectiveness of the standard Navy policy of using silver restorations vice gold restorations in the type of caries in which some portion of a cusp or cusps require a restoration, has been questioned. In this type of occurrence the restoration, silver or gold, is asked to take over the role of the cusp. By the inherit nature of the way gold and silver are applied as restorations, gold is considerably stronger and in fact, if gold is used as the restoration material, it is considered to be essentially permanent. On the other hand, a silver restoration has a much higher incidence of failure.

If a model of the process of cusp failure and restoration is determined and cost estimates for material and labor are computed, then the economic feasibility of using gold vice silver restorations may be considered.

B. BACKGROUND

The dentition for the adult human mouth consists of thirty-two teeth. The four first molars, or wisdom teeth, are often not included in the normal adult male dentition since for many adults these four never appear; and if they do, they do not function in the process of mastication. Figure 1 depicts the remaining twenty-eight teeth in an example of the natural dental arch, which is further divided into the maxillary (upper) arch and the mandibular (lower) arch with the midline of each represented by the straight line. In addition, each arch is divided into three sections as depicted by the wavy lines and numbered one through six. The

individual teeth are numbered one through thirty-two with the four wisdom teeth missing as previously noted. Each tooth in sections one, three, four, and six, which include the eight functional molars and eight bicuspids, has five surfaces which are the occlusal (O), or the biting surface of a tooth; the mesial (M), the side of the tooth closest to the midline of each arch; the distal (D), the side of the tooth farthest from the midline of each arch; the lingual (L), the side of the tooth closest to the tongue and the facial (F), the side of the tooth farthest from the tongue. The dental profession uses letters to represent the technical names of the surfaces for simplicity and the same procedure is followed in the Navy. Each of these sixteen teeth has from two to five cusps with the cusp being the highest part of the tooth above the gingiva (gum). Whenever the surface letter F or L is used in conjunction with any other tooth surface in describing a caries, it implies that one or more cusps are being replaced. Figure 2 illustrates the surfaces and the cusps while Figure 3 illustrates some basic and major types of caries that may occur, with the shaded areas representing the portion of the tooth that has had a caries and has been restored regardless of the restoration material used. Teeth numbered 30, 31, and 19 in Figure 3 and in the example dental health record in Appendix A are illustrations of teeth with cusp breakdown [Refs. 7 and 9].

Dental caries can attack any of the surfaces at any time during the course of an individual's life. However, formation of caries is not a biological aging process; rather it develops in the following manner. When an individual leaves food on his teeth after eating, the food is acted on by acids and enzymes which is the mouth's way of digesting. This process forms a substance known as plaque which then develops its own acids and these acids are what attacks the enamel of teeth, forming caries.

Dental caries, the most common disease known to man, can occur on a caries free surface, a surface with previous caries, or under an existing restoration on any surface, and they can occur in single or multiple areas. In the adult human mouth caries that involve cusp breakdown do not usually begin to appear until an individual is in his mid-twenties and is a more advanced stage of dental caries.

In considering cusp breakdown in the eight functional molars and eight bicuspids, the restoration materials that are used are silver and gold with the use of silver being standard Navy policy. The advantages of silver are that it is very inexpensive and takes less time than gold to install. The advantage of gold is that it is much more durable than any other material that is used. It generally takes twice as long to install gold as it does silver because of the method of installation. If a gold restoration is to be used, a cast is made for the restoration and it is then formed separately using molten metal. The forged restoration is then installed. A silver restoration is mixed similarly to cement and then installed with the setting of the material taking place on the tooth [Ref. 5].

C. SCOPE OF THE STUDY

It is the purpose of this study to investigate the cost effectiveness of the Navy policy of using silver restorations vice gold restorations in the type of caries which involve cusp breakdown. Because this type of occurrence does not usually appear until an individual is in his mid-twenties, this study will only be concerned with career military officers. Then, given that gold takes twice as long as silver to install, that gold costs considerably more than silver, that there is a positive probability that the silver restoration will have to be replaced and that only career

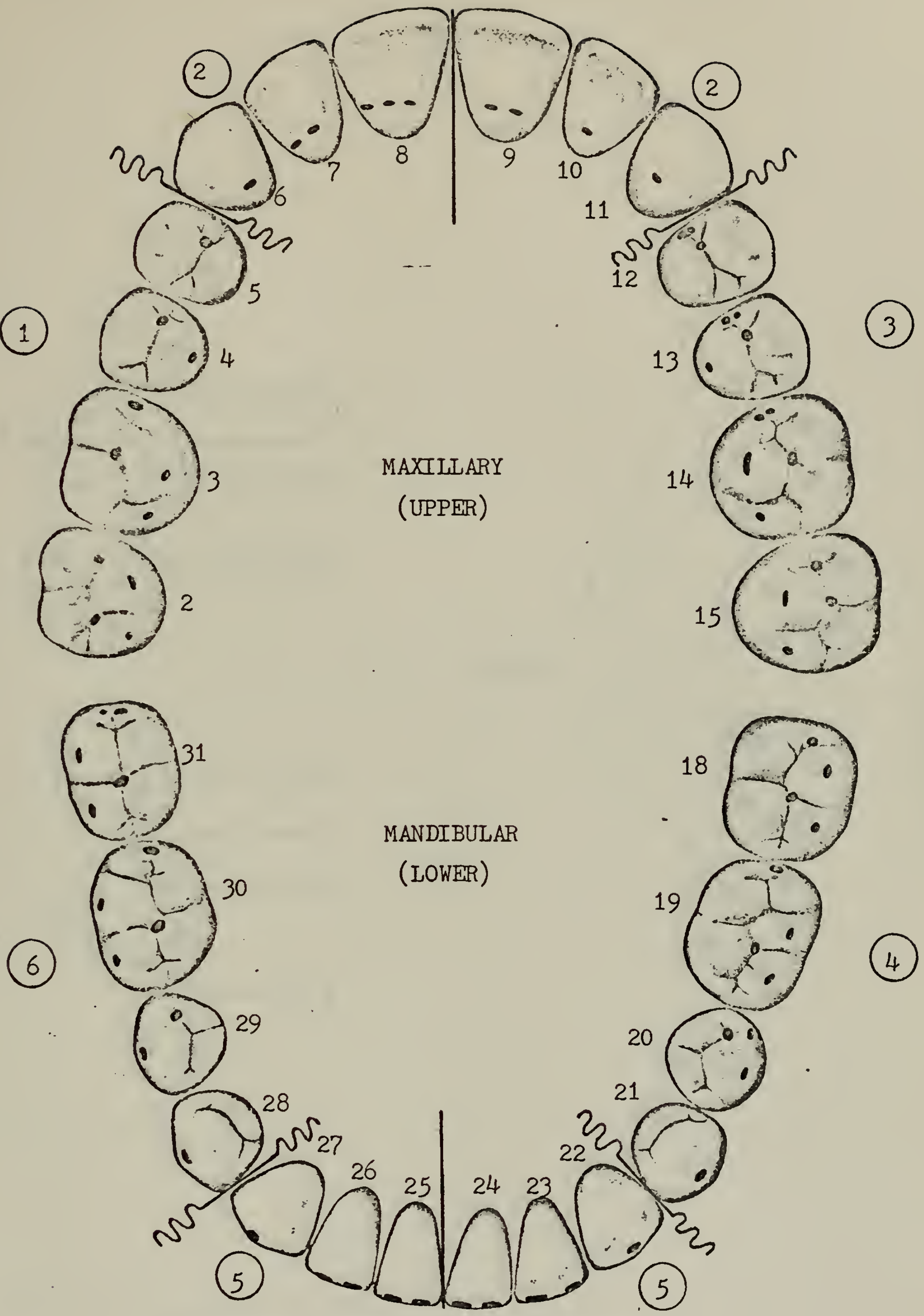


Figure 1. Normal Dental Arch

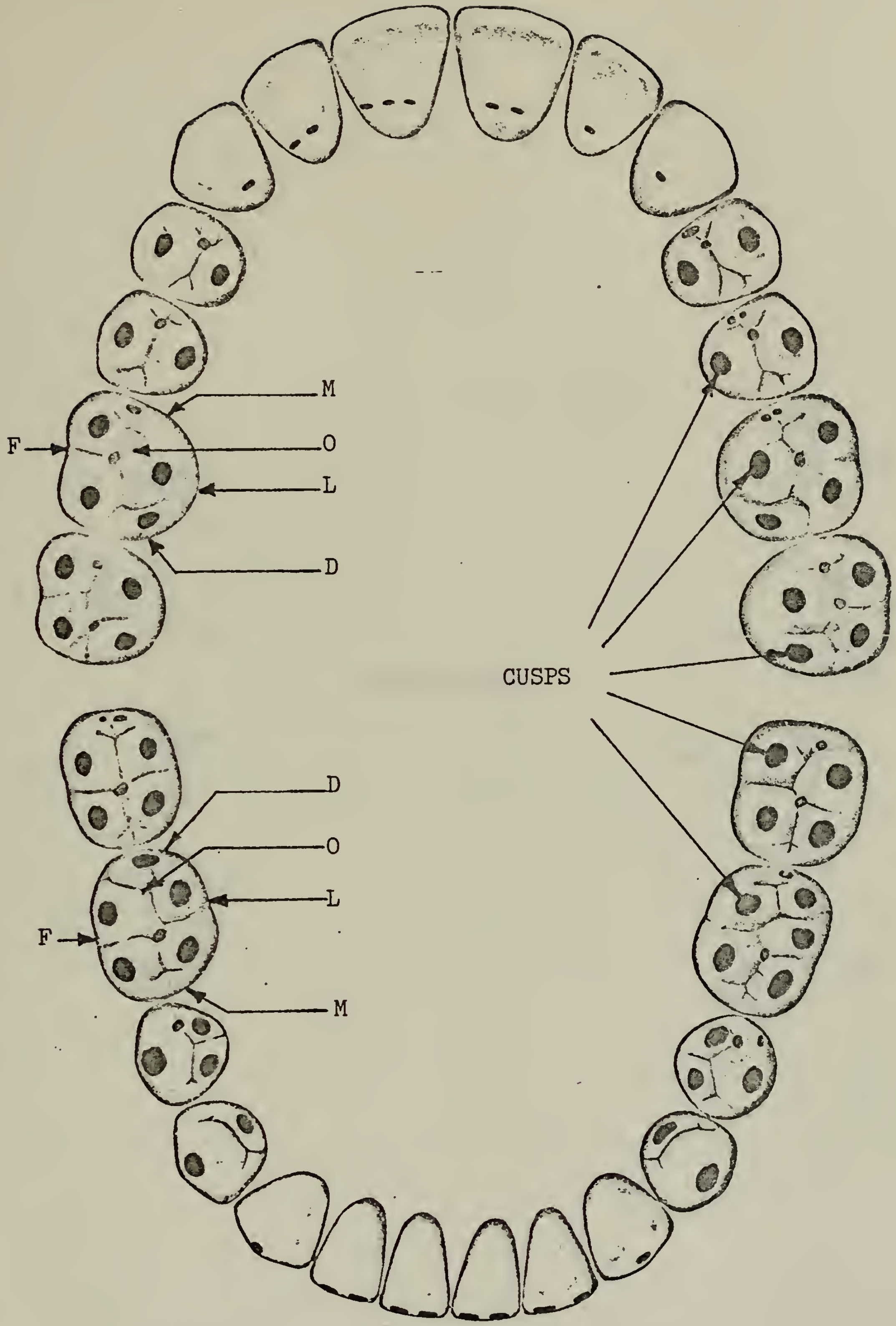


Figure 2. Cusps and Tooth Surfaces

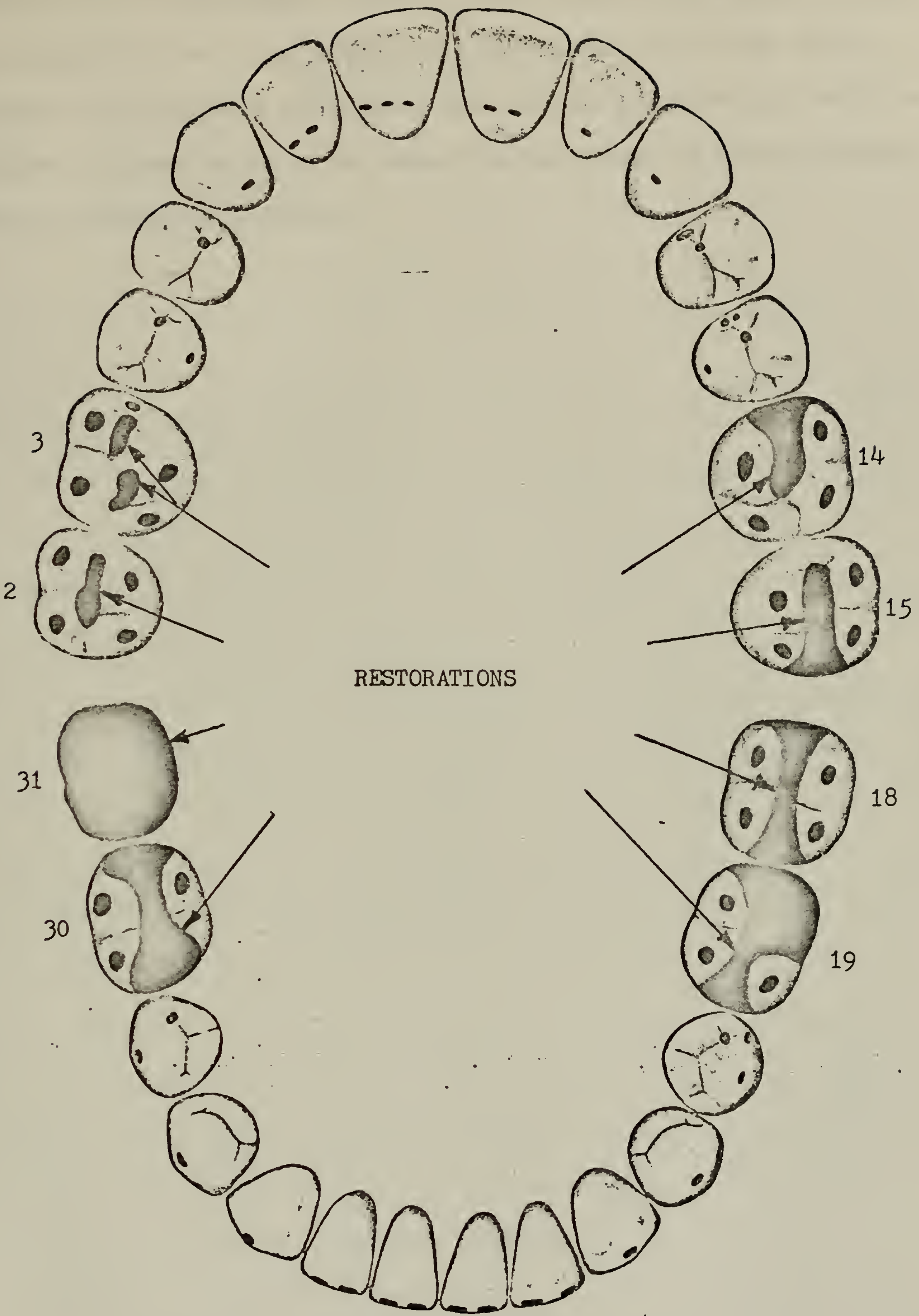


Figure 3. Sample Caries and Restorations

officers will be considered, Section II will present the empirical data or data base for the study, Section III will develop the model used to represent the occurrence and restoration process, and Section IV will use the model and cost estimates to obtain the basis for the results and conclusion presented in Section V.

II. DATA COLLECTION

The Naval Postgraduate School has an assigned dental billet of five dentists. At the present time this billet is manned by one Lieutenant (LT), one Lieutenant Commander (LCDR), one Commander (CDR), and one Captain (CAPT). There are also nine dental technicians and one dental hygienist. The department is responsible for the care of approximately 1400 officer students and 200 staff officers. In addition, they are responsible for all Naval retired personnel within a radius of forty-five miles on a space available basis. The retired personnel are limited to certain types of care, and their records are not kept on file in the Dental Department. The Dental Department is responsible for the maintenance of the dental records of all active personnel assigned to the Postgraduate School. The sample data was taken from the dental histories of these active duty military officers.

In an attempt to randomize the total sample size of 500 officers, the data was collected in the following manner. Twenty-two officers were selected from each of the letters A through T, twelve each from U and V, two each from X, Y, and Z, and thirty from W. Within each letter, samples were chosen in alphabetical order. It is recognized that certain letters of the alphabet may contain a majority of individuals with a common background, but this question was ignored except for the letters U, V, X, Y, and Z where limited numbers required a smaller selection.

For each of the eight functional molars and eight bicuspids of each officer that had any dental history, a separate numerically coded computer entry was recorded. This resulted in 2945 teeth being recorded as a data sample. The items recorded for each tooth were:

The rank or pay grade of the officer.

The birth date of the officer.

The officer's date of entering military service.

The identifying number of the tooth.

The tooth's dental history at the time of entering service by type of occurrence and type of restoration.

The tooth's complete in-service dental history by type of occurrence, date of occurrence, and type of restoration.

All dates were recorded as month and year. Appendix B contains the format used for coding this data.

Data on the cost of the two materials, silver and gold, was obtained from the Postgraduate School Dental Department and Reference 2 provided the basis for computing labor costs. On entering military service, a dentist is given a professional date for pay and rank purposes. This amounts to an allotment of four years service time for four years in dental school. In those cases where a dentist has worked as a civilian dentist, he is also allotted an additional service time computed as three-fourths times the years worked as a civilian dentist. For this study only the four year allotment will be considered. There are two sources of special pay or incentive pay for military dentists. The first is incremented on an increasing basis depending on years served and computed using the professional date, and the second is a yearly lump sum payment, continuation pay, that is incremented on an increasing basis through the rank of CAPT and begins after serving five years of continuous active duty. Continuation pay does not use the professional date for computation. Enlisted members of a dental staff do not draw any special pay privileges.

Reference 3 provides the information that there are 1867 Navy Dentists: 1060 LT's, 304 LCDR's, 273 CDR's, 225 CAPT's, and five Rear Admirals

currently in the Navy, and that there are less than ten LCDR's who do not and no LT's who do qualify for continuation pay.

III. DESCRIPTION OF THE METHOD AND ASSUMPTIONS

A. MODEL ASSUMPTIONS

For each tooth in the sample that had occurrences of the type in question, or cusp breakdown, the following time sequence of events was considered. The first in service occurrence was denoted as time zero for the sequence and the time between the i^{th} and the $i^{th} + 1$ occurrence is X_i, $i=1,2,\ldots n$. The sequence was terminated at the time the data sample was taken and the time between the last occurrence and termination is X_f. The time from entering military service to the first in service occurrence is Y_1 if there has been a before service occurrence and Y_2 if there has not been one. Any occurrence before an individual entered military service with no further in service occurrences were not considered since they entailed no cost to the military. Figure 4 depicts the time sequence.

In order for this particular problem to be appropriately modeled by a poisson process, the validity of certain assumptions must be addressed. These assumptions involve the stationarity and independence of the stochastic process associated with the time sequence of occurrences. Their validity is supported on theoretical grounds from the following. This type of caries, or cusp failures, does not usually begin to appear until an individual is in his mid-twenties. However, as previously noted, this sequence does not occur because of biological aging but is rather directly related to the type of dental care that has been given to the teeth. Therefore, since occurrences depend only on the formation of plaque or sudden stress biting situations, it can be assumed that the sequence is stationary. Since the sequence begins with a failure and consequently a restoration, any further occurrences, once restored, are considered to

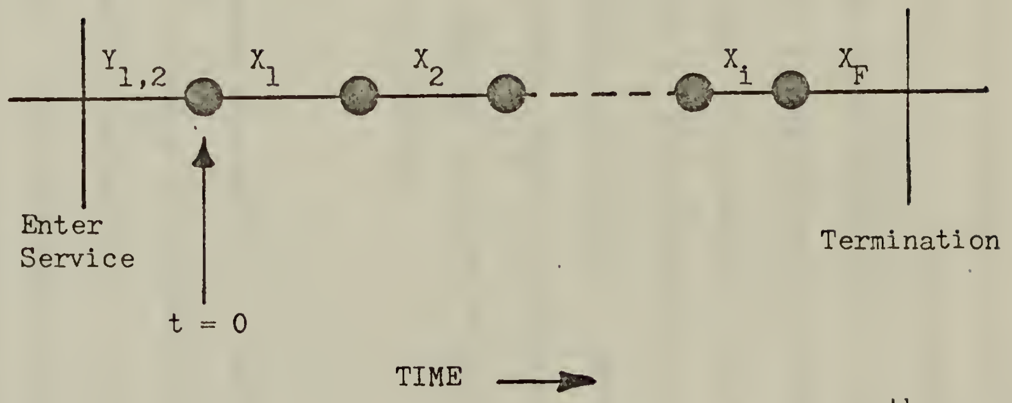


Figure 4. Time Sequence of Occurrences

X_i = i^{th} Interoccurrence Time

X_F = Termination Interoccurrence Time

Y_1 = Time in Service to First Occurrence if there has been a before service occurrence

Y_2 = Time in Service to First Occurrence if there has been no before service occurrence

$i = 1,2,\ldots$

be as strong as the original restoration and it can therefore be assumed that the interoccurrence times are independent. Statistical support for the validity of these assumptions is considered in Section IV. On the basis of these statistical and theoretical considerations, independent identically distributed (IID) interarrival times are assumed.

The assumption is also made that after one occurrence on an individual tooth all future occurrences on that tooth will be of the same type. The data shows this to be a valid assumption.

If there was an occurrence for an individual before he entered military service, it is recorded only as having occurred with no time reference. Consequently, it is assumed that an occurrence before service will not have an affect on the distributions of occurrences that take place while an individual is in service. Because of the age of an individual when the caries first begin to take place, one would expect that the number of these type of occurrences would be small. This was in fact seen to be true in the data.

B. THEORY

Let $N(t)$ denote the number of occurrences by some time t, $t \geq 0$. For $i \geq 1$, let X_i denote the elapsed time between the i^{th} and $i^{th} + 1$ occurrences, then the sequence X_i, $i = 1,2,\ldots$ is a sequence of interoccurrence times. If, in addition, these interoccurrence times are independent identically distributed exponential with parameter λ, and the counting of occurrences begins at time zero, then the time sequence is stationary, and the number of occurrences in any interval of length t is poisson distributed with mean λt, and the sequence of occurrences may be modeled by a poisson process [Refs. 1 and 10].

If for a sequence of IID exponential interoccurrence times, X_f denotes the amount of time from the last occurrence until a termination time t_f, then, from renewal theory, the distribution of these times is just a censored exponential with parameter λ. In addition, if the time $T = (0, t_f]$ is large relative to $1/\lambda$, the limiting distribution of X_f is exponential with parameter λ [Refs. 1 and 10].

IV. DATA PROCESSING

A. MODEL DERIVATION

From the 2945 sample teeth, there were 333 that had at least one in service occurrence of the type in question. Of these 333, there were only 14 that had had an occurrence before entering service. There were no cases of an occurrence before service that did not have any occurrences in service. Of the 333, there were:

97 teeth that had at least two occurrences
32 teeth that had at least three occurrences
10 teeth that had at least four occurrences
2 teeth that had at least five occurrences

There were 165 individuals out of 500 that had at least one tooth with an occurrence of the type in question. Of these 165, the following was noted:

4 individuals had 7 teeth with at least one occurrence
4 individuals had 6 teeth with at least one occurrence
7 individuals had 5 teeth with at least one occurrence
11 individuals had 4 teeth with at least one occurrence
15 individuals had 3 teeth with at least one occurrence
33 individuals had 2 teeth with at least one occurrence
91 individuals had 1 tooth with at least one occurrence

Also, for these 165 individuals:

The average time in service = 10.81 years
The average age = 33.33 years
The average rank or pay grade = 03.43

The average time in service before the first occurrence per tooth of an individual = 74.57 months

For the total sample of 500:

The average time in service = 9.51 years

The average age = 31.82 years

The average rank or pay grade = 03.28

Reference 6 provided a modified two-sample Kolmogorov-Smirnov computer program that is being prepared for the Naval Postgraduate School Computer Center Library. This program was used to test the hypothesis that the distribution of the interoccurrence times between the first and second occurrence was equal to the distribution of the interoccurrence times between the second and third occurrence. As a result of this test, the hypothesis of no difference in the two distributions of interoccurrence times was accepted at a high (>.50) level of significance. The sample sizes for the distributions between the second and third occurrences and between the third and fourth occurrences were too small to test and were assumed to have the same distribution as those of the hypothesis, or $F_{X_1}(x) = F_{X_2}(x) = F_{X_3}(x) = F_{X_4}(x)$.

From histograms cumulatively plotting the 141 X_i interoccurrence times plus the 333 truncated interoccurrence times versus frequency [Figure 5] and the 141 X_i versus frequency [Figure 6] it appeared that an exponential distribution may model the distribution of the interoccurrence times. A maximum likelihood estimation procedure was used to find an estimate for the parameter of the exponential distribution. Since independent and stationary increments have been assumed and the process is truncated, the likelihood function for an exponential distribution is

$$L(\lambda) = f_{X_1}(t) \cdot f_{X_2}(t) \text{----} \cdot [1-F_{X_f}(t)]$$

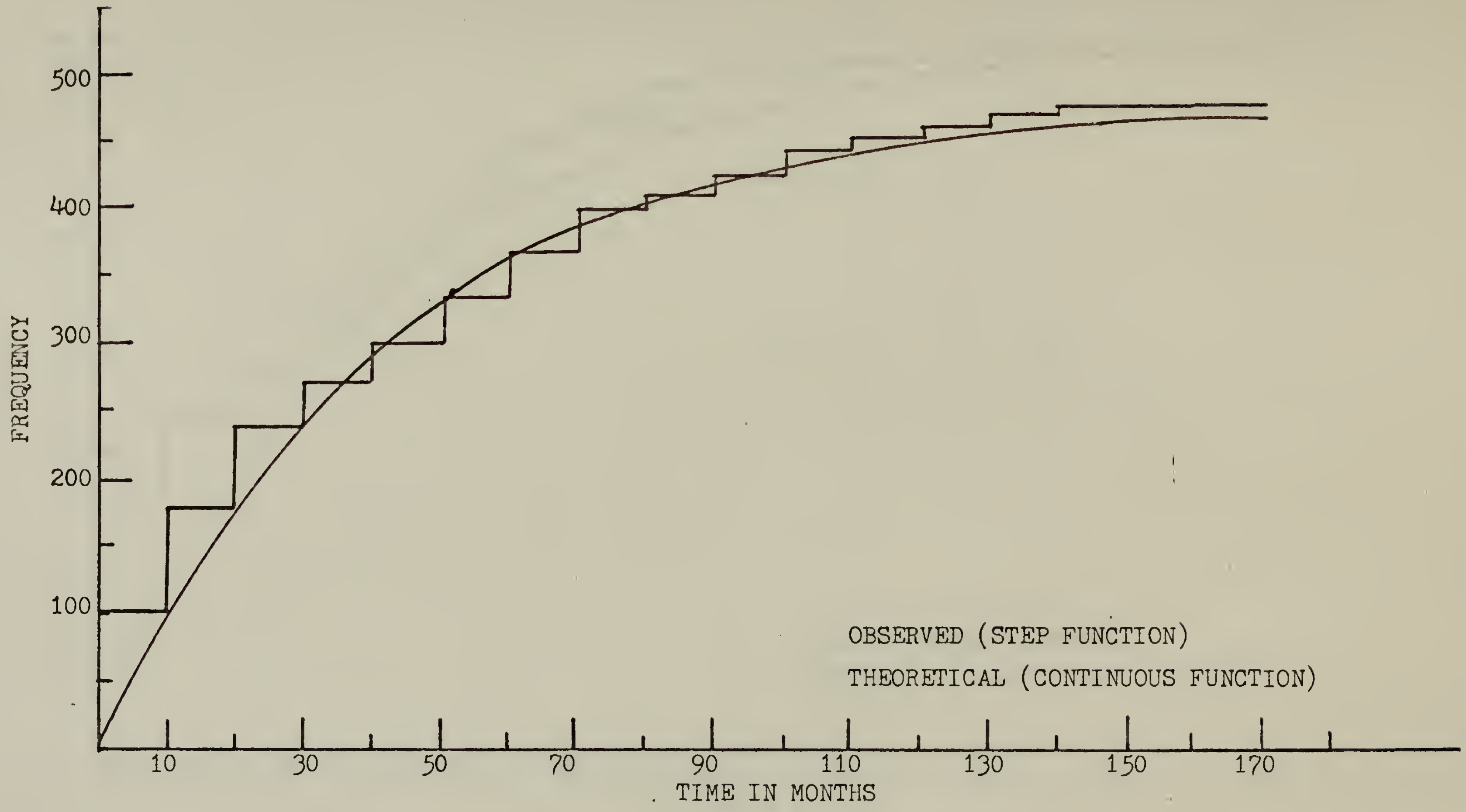
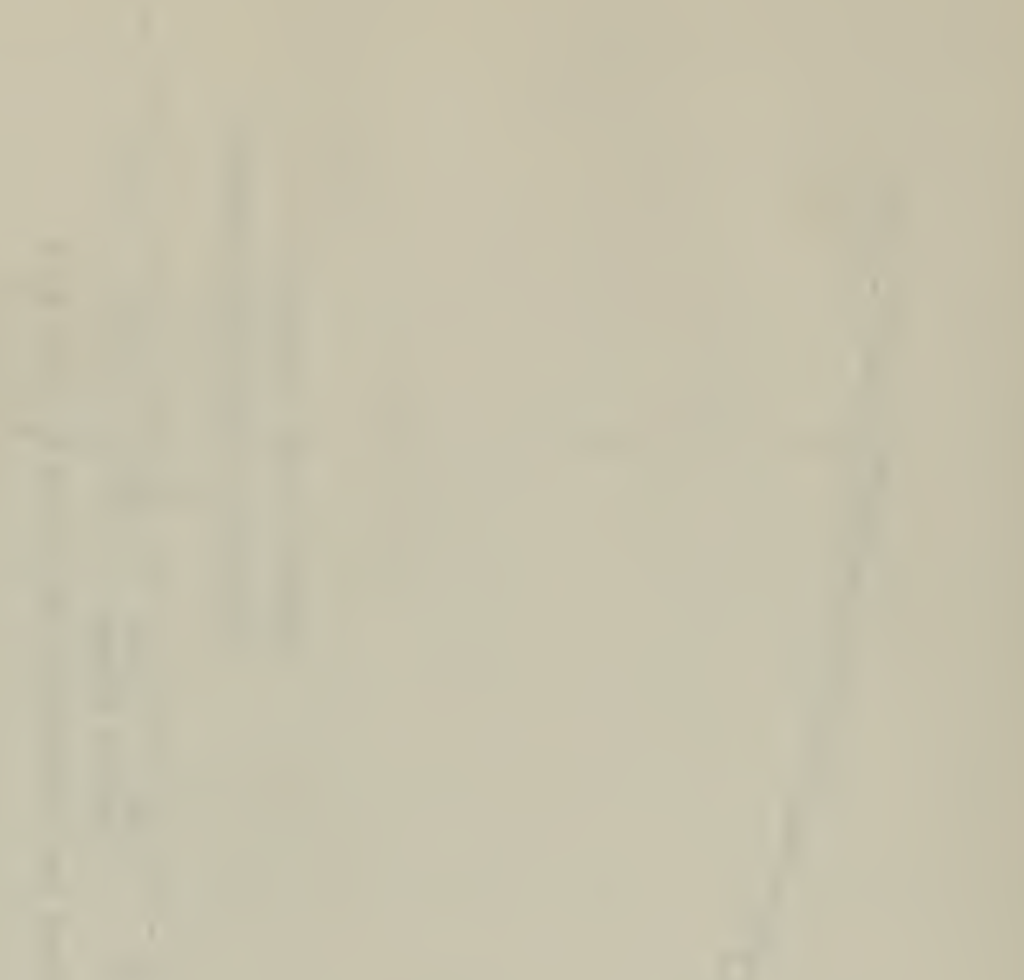


Figure 5. Observed and Theoretical CDF of the X_i's and X_f's



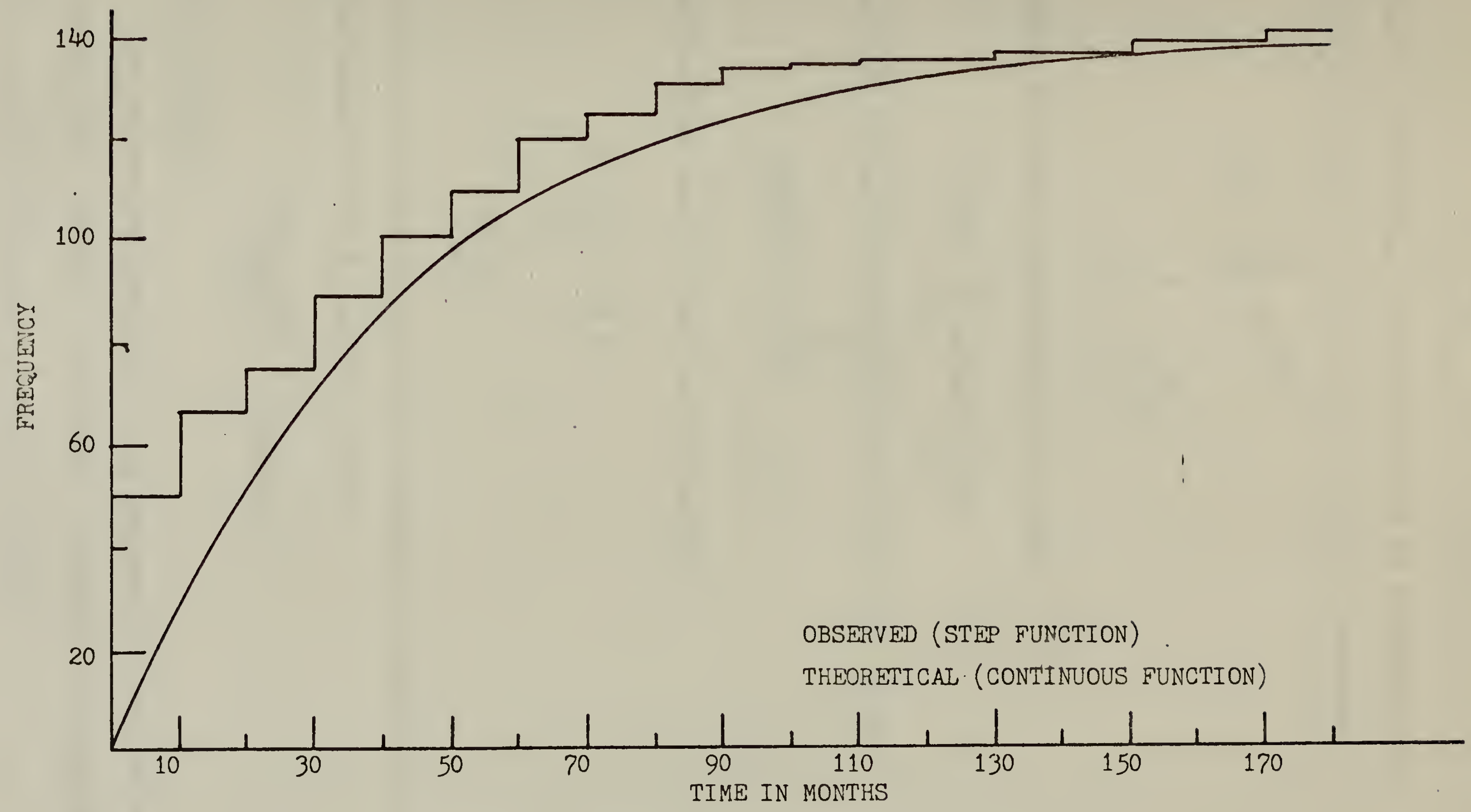


Figure 6. Observed and Theoretical CDF of the X_i's

$$= \lambda e^{-\lambda t_1} \cdot \lambda e^{-\lambda t_2} \text{---} \cdot \lambda e^{-\lambda t_n} \cdot e^{-\lambda(T-t_1-t_2\text{---}-t_n)}$$

where T is the total in service time of an individual tooth. Letting $T_i=(t_o,T)$, where t_o is the time of the first occurrence, it follows that

$$L(\lambda) = \prod_{i=1}^{n} \lambda^n e^{-\lambda T_i}$$

$$= \lambda^{\sum_{i=1}^{n}} e^{-\lambda \sum_{i=1}^{n} T_i}.$$

Taking the natural logarithm of both sides

$$\ln L(\lambda) = \sum_{i=1}^{n} n \ln \lambda - \lambda \sum_{i=1}^{n} T_i$$

and then taking the derivative with respect to λ gives

$$\frac{d \ln L(\lambda)}{d \lambda} = \frac{\sum_{i=1}^{n} n}{\lambda} - \sum_{i=1}^{n} T_i.$$

Setting this equal to zero and solving for λ

$$0 = \frac{\sum_{i=1}^{n} n}{\lambda} - \sum_{i=1}^{n} T_i$$

$$\lambda = n / \sum_{i=1}^{n} T_i$$

which is an estimate for λ of the exponential distribution. Substituting the values for 141 X_i and 333 X_f gives

$$\lambda = \frac{141 + 333}{20129} = .02355/\text{month}$$

as an estimate.

The theoretical frequency of an exponential distribution was calculated using this estimate. Then using a standard Chi-Squared goodness of fit test with m intervals, the test statistic

$$\sum_{i=1}^{m} \frac{(O_i - T_i)^2}{T_i} ,$$

where O_i = the observed or sample frequency in interval i and T_i = the theoretical frequency in interval i, was calculated for the distribution of the X_i and for the distribution of the X_i and X_f combined. To test the hypothesis that the sample distribution was equal to the theoretical distribution, the test statistic for both cases was compared with a Chi-Squared table using m-2 degrees of freedom. The result in both cases was that the hypothesis that the distributions were equal was accepted only at a low ($<.05$) level of significance [Refs. 4 and 8].

Looking at $(O_i - T_i)$ for each interval to find the greatest sources of deviation from the theoretical model, two intervals, 100 to 110 months in the distribution of X_i and X_f combined and zero to ten for the distribution of X_i, gave the largest deviations. A possible source of deviation for the 100-110 interval is that if one individual had several teeth with occurrences of the type in question, it may influence a particular interval if they had been worked on at the same time. The data showed, however, that this was not the case. There was no explanation found for the large observed deviation in the interval of 100 to 110.

In considering the 0-10 interval, it was found that from the 474 observed interoccurrence times there were twenty-one cases where zero time (less than one month between occurrences) was recorded. Five of these were from truncations and sixteen were recorded as times between occurrences. The probability of that short a time between occurrences is very low. So the possibility that other than normal circumstances were influencing the zero times was considered. According to dental advisors there is a high probability that this short time was caused by a restoration that did not set properly or by other abnormal situations.

Consequently, the possibility of a mixed distribution was considered and the following approach was taken. Dropping the sixteen less than one month interoccurrence times and calculating, as before a new λ estimate equal to .022753/month and again using a Chi-Squared goodness of fit test, then the result was that the hypothesis that the distribution of the X_i's equalled the theoretical distribution was accepted at a high (>.50) level of significance. The hypothesis that the X_i and X_f distribution equalled the theoretical distribution was accepted at a low (<.10) level of significance. The unexplained large deviation in the 100 to 110 month interval is still the major factor in the latter distribution. Assuming this deviation to be the result of a random occurrence, the λ estimate of .022753/month will be used as the parameter of an exponential distribution to model the interoccurrence times. Since each tooth is considered separately, this means that there are λt events over a given interval t. This, along with the independent and stationary increment assumptions, allow the time sequence of occurrences to be modeled by a poisson process.

However, the sixteen zero points must also be considered since they are valid sample interoccurrence times. An estimate of the probability of an observed interoccurrence time being one of these 16 is $p = 16/474 = .0338$. Then the probability of the number of this type of failure occurring out of a certain number of potential occurrences can be modeled as a binomial random variable with parameters n,p where n is the total number of poisson failures and p is the estimate of the probability of a binomial failure.

From the examination of data there were no occurrences in which two zero time occurrences followed each other. Consequently, the assumption is made that there cannot be two consecutive occurrences with zero

interarrival times. With this assumption and remembering that the process begins with an occurrence with probability one, then the following mixed distribution model for the process was developed.

Letting V represent a binomial occurrence, $\overline{V}$ no binomial occurrence, W and $\overline{W}$ respective occurrence and no occurrence for the poisson process and X the number of occurrences for the mixed distribution process, then

$$
\begin{aligned}
P(X=1) &= P(\text{no more occurrences}) \\
&= P(\overline{V})P(\overline{W})
\end{aligned}
$$

and

$$
\begin{aligned}
P(X=2) &= P(\text{one more occurrence}) \\
&= P(V)P(\overline{W}) + P(\overline{V})P(W) \\
P(X=3) &= P(\text{two more occurrences}) \\
&= P(V)P(W)P(\overline{V}) + P(\overline{V})P(W)P(V) + P(\overline{V})P(W)P(\overline{V})P(W)P(\overline{V}) \\
&= 2\, P(V)P(W)P(\overline{V}) + P(2W)P(3\overline{V}) \\
&\vdots
\end{aligned}
$$

continuing in this manner and remembering that

$$
\begin{aligned}
P(X > x) &= 1 - P(X \leq x) \\
&= 1 - [P(X=0) + P(X\ 1) + \text{---} + P(X=x)]
\end{aligned}
$$

the following general model is derived.

$$
P(X > x) = 1 - \sum_{i=1}^{x} \sum_{j=0}^{x} \frac{e^{-\lambda t}(\lambda t)^j}{j!} \cdot \binom{j+1}{i-1-j} P^{i-1-j} q^{2j+2-i}
$$

$$
j \geq \left[\frac{i-1}{2}\right]
$$

where i represents the total number of occurrences which happen, one of which is the initial occurrence and of the i-1 remaining failures, there are j poisson events and i-1-j binomial events.

B. RESTORATION COST

With a model for the process it was next necessary to consider the cost of doing gold and silver restorations. To find a total cost figure, three areas are considered; the cost of materials, the cost of labor, and the cost of time lost by the patient. From these the point at which the cost of repeated silver restorations becomes larger than gold ones can be determined.

The cost for the restoration materials silver and gold were computed in the following manner. Silver is purchased in 400 pellet lots and the present price, which has been relatively stable for several years, is $6.84 for 400 pellets. Gold is purchased by penny-weight with twenty penny-weights in an ounce. One year ago gold was $3.00 for a penny-weight but is presently selling for $7.16 a penny-weight. An average restoration uses four to five pellets of silver or four to six penny-weights of gold. Using an average of five pellets or five penny-weights for a restoration implies a cost of $.085 for a silver restoration and $35.80 for a gold restoration.

Labor costs were computed from the following. A silver restoration requires one hour of time from one dentist and one chair side assistant for installation. The installation of a gold restoration requires two hours time from one dentist and one chair side assistant and in addition requires four hours time from a laboratory technician. Two cases were considered for the enlisted assistants. First, the chair side assistant was considered to be in the pay grade E4 with over two years service and the laboratory technician was considered to be in pay grade E5 with over three years service. The second case was with an E4 over three years chair side assistant and an E5 over four years laboratory technician. Several officer, or dentist, combinations were considered but the following

will be presented and carried through the subsequent discussion. A Lieutenant with over six years, a Lieutenant Commander with over ten years, a Commander with over fourteen years, and a Captain with over eighteen years. The ranks correspond to pay grades 03, 04, 05, and 06, respectively. The average pay grade of 03.43 and the average time of service of 10.8 years of the sample for those with occurrences of the type in question were used to compute an average hourly cost for the patients lost working time. No special pay of any kind was considered for the patient. The pay was then computed as .57 (Lieutenant over ten) + .43 (Lieutenant Commander over ten).

For a one year period it was assumed that there are 52 weeks per year with five eight hour work days per week. Considering that everyone is allowed 30 days leave per year and arbitrarily assuming that an individual will miss 22 work days per year, it is calculated that the average person works 1200 hours per year. This obviously does not take into account duty periods, emergencies and other such items but since the dental department has a relatively regulated work schedule, they will be ignored. For the computation of total pay no deductions were considered and the basic allowance for quarters was figured for married personnel only. From reference 2 and computation the following was obtained.

Table I. Dental Officer's Monthly Pay

	Base	Incentive	Continuation	Subsistence	Allowance for Quarters	Total Monthly
LT	1036.50	250.00	0.0	47.88	195.60	1529.98
LCDR	1187.70	350.00	395.90	47.88	215.40	1800.98
CDR	1388.40	350.00	462.80	47.88	238.80	2025.08
CAPT	1759.20	350.00	586.40	47.88	258.30	2415.38

Table II. Enlisted Technician's Monthly Pay

	Base	Allowance for Quarters	Total Monthly
E5 over 4	458.10	138.60	596.70
E5 over 3	438.90	138.60	577.50
E4 over 3	413.10	121.50	534.60
E4 over 2	390.60	121.50	512.10

Table III. Pay by Hour

	Officer		Enlisted
LT	15.30	E5 over 4	5.97
LCDR	18.01	E5 over 3	5.78
CDR	20.25	E4 over 3	5.35
CAPT	24.15	E4 over 2	5.12

Table IV. Cost of Restoration versus Rank of Dentist

	LT	LCDR	CDR	CAPT
Gold Restoration	129.12 127.90	142.44 141.22	148.26 147.04	158.56 157.34
One Silver Restoration	34.80 34.58	41.47 41.24	44.38 44.15	49.53 49.30
Two Silver Restorations	69.60 69.16	82.94 82.48	88.76 88.30	99.06 98.60
Three Silver Restorations	104.40 103.74	124.41 123.72	133.14 132.45	148.59 147.90
Four Silver Restorations	139.20 138.32	165.88 164.96	177.52 176.60	198.12 197.20
Five Silver Restorations	174.00 172.90	207.35 206.20	221.90 220.75	247.65 246.50
Six Silver Restorations	208.80 207.48	248.82 247.44	266.28 264.90	297.18 295.80

The top line in each block represents an E5 over 4 and an E4 over 3. The bottom line in each block represents an E5 over 3 and an E4 over 2.

From Table IV it can be seen that within each rank the cost of putting in four silver restorations exceeds the cost of putting in a gold one. This is true for other combinations of the pay scales also. There are combinations such as a comparison of a Captain doing a gold and a Lieutenant doing the silver restorations, or a Lieutenant doing a gold and a Captain doing the silver restorations, in which the cost of silver restorations exceeds the cost of a gold one at five silver restorations and three silver restorations, respectively. Other combinations give similar results but within a pay grade it remains that the cost of four silver restorations exceeds the cost of a single gold one.

C. APPLICATION

For a career officer, service times for retirement of twenty, twenty-three and twenty-five years were considered. For the moment assuming that twenty years is the service time to retirement, then to find the average time in service after the first occurrence subtract the average time in service until the first occurrence from twenty years, or

$$20 \text{ yr} - \frac{74.571/\text{mo}}{12 \text{ mo/yr}} = 13.786 \text{ yrs} = t$$

Then using the estimated value for λ previously developed the mean of the poisson process is

$$\begin{aligned}\lambda t &= (.02275 \text{ occurrences/mo})(13.786 \text{ yrs})(12 \text{ mo/yr}) \\ &= 3.764 \text{ occurrences}\end{aligned}$$

Since the process starts with an occurrence, the expected number of poisson events is $\lambda t + 1 = 4.764$ occurrences. As previously noted, the cost of four silver occurrences exceeded the cost of a gold restoration within a rank. Therefore, with the number of occurrences for a tooth equal to X and using λt, p and the mixed model

$$P(X=i) = \sum_{j=0}^{i-1} \frac{e^{-\lambda t}(\lambda t)^{j}}{j!} \cdot \binom{j+1}{i-1-j} p^{i-1-j} q^{2j+2-i}$$

$$j \geq \left[\frac{i-1}{2}\right]$$

For example

$$P(X=4) = \sum_{j=1}^{3} \frac{e^{-\lambda t}(\lambda t)^{j}}{j!} \cdot \binom{j+1}{3-j} p^{3-j} q^{2j-2}$$

$$= .19637.$$

If similar computations are made for i=1,---x, then the following can be calculated.

$$P(X\geq 4) = 1 - [P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - \sum_{i=0}^{4} \sum_{j=0}^{i-1} \frac{e^{-\lambda t}(\lambda t)^{j}}{j!} \cdot \binom{j+1}{i-1-j} p^{i-1-j} q^{2j+2-i}$$

$$j \geq \left[\frac{i-1}{2}\right]$$

$$= 1 - .002253 + .08273 + .15473$$

$$= .740.$$

This implies that there is approximately a 74% chance that once a career officer has an occurrence, he will have four or more occurrences if silver restorations are used.

In addition, one may compute the expected cost using the expected number of restorations for the selected times in service for a career officer and expected cost per restoration based on the proportion of each dental rank from the total current Navy dentist population. The following procedure is used to calculate expected cost.

The expected number of occurrences is

$$E[X] = 1 \cdot P(X=1) + 2 \cdot P(X=2) + 3 \cdot P(X=3) + --- x \cdot P(X=x) + ---.$$

For computational reasons, the calculations were stopped at X=9. As a result

$$E[X] \geq 1 \cdot P(X=1) + \text{---}\ 9 \cdot P(X=9)$$

$$= 5.0270$$

which implies that, given a tooth has had an occurrence, the mean number of occurrences for that tooth in an individual with a twenty year service career is at least 5.027 occurrences. The expected number of occurrences for the mixed distribution should be larger than $\lambda t+1$ of the poisson process, but since computations were truncated at X=9, the conservative estimate of $\lambda t+1$ is used as the expected number of occurrences in computing expected cost, remembering that the actual value of $E[X]$ would be larger.

With 1862 Naval dentists in the rank of Lieutenant through Captain, the expected cost of a silver restoration is calculated in the following manner.

$$E[\text{cost}] = E[X] \cdot (\tfrac{1060}{1862} \cdot \text{cost of one silver restoration by a LT} +$$

$$(\tfrac{304}{1862} \cdot \text{cost of one silver restoration by a LCDR} +$$

$$(\tfrac{273}{1862} \cdot \text{cost of one silver restoration by a CDR} +$$

$$(\tfrac{225}{1862} \cdot \text{cost of one silver restoration by a CAPT}).$$

Substituting costs from Table IV using an E5 over 4 and an E4 over 3

$$E[\text{cost}] = 4.764\ [\tfrac{1060}{1862}\ (\$34.80) + \tfrac{304}{1862}(\$41.47) + \tfrac{273}{1862}\ (\$44.38)$$

$$+ \tfrac{225}{1862}\ (\$49.53)]$$

$$= \$186.13 \text{ for silver restorations.}$$

$E[X]$ for gold is equal to one and substituting the corresponding costs from Table IV

$$E[\text{cost}] = 1 \cdot [\tfrac{1060}{1862}(\$129.12) + \tfrac{304}{1862}(\$142.44) + \tfrac{273}{1862}(\$148.26)$$

$$+ \tfrac{225}{1862}(\$158.56)]$$

$$= \$137.66 \text{ for a gold restoration.}$$

Making similar computations for twenty-three and twenty-five year career service times the following is tabulated.

Table V. Probability of Occurrence, Cumulative Probability of Occurrence, and Expected Number of Occurrences Versus Years to Retirement

	-- Average years to retirement		
	20	23	25
λt	3.764	4.533	5.129
$\lambda t+1$	4.764	5.583	6.129
$P(X=1)$	.02253	.00984	.00572
$P(X=2)$	.08273	.04389	.02954
$P(X=3)$	.15473	.09948	.07222
$P(X=4)$	.19637	.15251	.12343
$P(X=5)$	.19060	.17784	.16024
$P(X=6)$	.14952	.16816	.16845
$P(X=7)$	.10159	.13423	.17631
$P(X=8)$	.09522	.09300	.11479
$P(X=9)$	.02959	.05706	.07805
$P(X\geq 3)$	.895	.946	.965
$P(X\geq 4)$	.740	.847	.892
$P(X\geq 5)$	.545	.694	.769
$P(X\geq 6)$	.353	.516	.609
$E[X]$	5.0270	5.1014	5.4420

Table VI. Expected Cost versus Years to Retirement

	Expected Cost for	
	Silver	Gold
20 year Retirement	186.13	137.66
	185.08	136.44
23 year Retirement	218.13	137.66
	216.90	136.44
25 year Retirement	239.46	137.66
	238.11	136.44

The top line in each block represents an E5 over 4 and an E4 over 3. The bottom line in each block represents an E5 over 3 and an E4 over 2.

Since $\lambda t+1$ was used for the expected number of occurrences, the results in Table VI are conservative.

V. RESULTS AND CONCLUSIONS

Assuming that the average service life of a career officer is a minimum of twenty years, then the expected cost of silver and gold restorations indicate that it would be more economical to use gold restorations than silver restorations. Additionally, as service life increased, the expected cost of doing silver restorations becomes increasingly larger than gold restorations.

Historical records were not consulted concerning the average service life of career officers, but it is suspected to be considerably over twenty years. An influence for the future is the concept of an all volunteer military force which may affect career patterns; and if studies were to show that retired personnel were taking significant advantage of dental care, it would increase the service life by a considerable amount. One difficulty is determining who is a career officer. If this is not possible, then there is a need to study the resulting costs of having gold installed as a policy in all officers who have occurrences of cusp failures.

Two mitigating circumstances may temper the results. There were 2945 teeth that had dental work of some kind, and these did not include all teeth in the human mouth. Of these 2945, only approximately 10% were of the type in question. Also, Lieutenants and Lieutenant Commanders outnumber Commanders and Captains by three-to-one. These factors may influence who does the silver or gold restorations and in what proportion.

However, reiterating, it appears to be more economic to do gold restorations rather than silver restorations in occurrences of cusp breakdown.

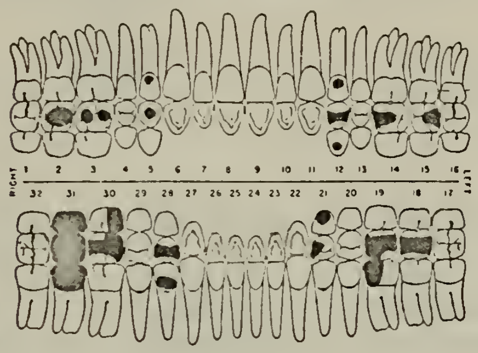
Standard Form 603-A
Promulgated December 1958
By Bureau of the Budget
Circular A-32 (Rev.).

APPENDIX A

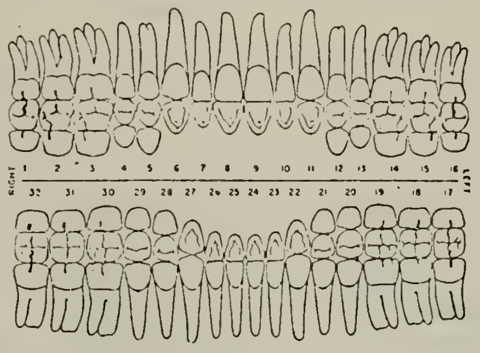
HEALTH RECORD | **DENTAL—Continuation**

SECTION III. ATTENDANCE RECORD

15. RESTORATIONS AND TREATMENTS (*Completed during service*)



16. SUBSEQUENT DISEASES AND ABNORMALITIES



REMARKS

REMARKS

17. SERVICES RENDERED

DATE	DIAGNOSIS—TREATMENT	CLASS	OPERATOR AND DENTAL FACILITY	INITIALS
	The date is given as day, month, year.			
	The format for listing caries is as follows:			
	#2 - O			
	#3 - O,O			
	#5 - O,F			
	#14 - MO			
	#15 - DO			
	#18 - MO			
	#19 - MODF			
	#30 - MODL			
	#31 - MODFL			
	#28 - MOD,F			
	#21 - MO,L			
	#12 - MOD,F,L			

PATIENT'S LAST NAME - FIRST NAME - MIDDLE NAME

IDENTIFICATION NO.

APPENDIX B. Format of Data Cards

Col 1-3	Col 4	Col 5-8	Col 9-10	Col 11-20	Col 21-30
Sample# 001-999	Rank 1-ENS 2-LTjg 3-LT 4-LCDR 5-CDR 6-CAPT	Initial Date of Record	Tooth #	Occurrence Before Military Service	First In Service Occurrence

Col 31-40	Col 41-50	Col 51-60	Col 61-70	Col 71
Second In Service Occurrence	Third In Service Occurrence	Fourth In Service Occurrence	Fifth In Service Occurrence	# of Occurrence of First MOD

Col 72	Col 73	Col 74	Col 75-78
Out of Total # of Occurrences	# of Occurrence of First Cusp Restoration	Out of Total # of Occurrences	Birth Date

Columns 11-20 through columns 61-70 are in the following format.

Restorations Date

The restoration codes are two digit designators. There may be as many as three codes for any one occurrence. All dates are listed as two digits for a month and two digits for a year.

Restoration Codes

M-01
O-02
D-03
F-04
L-05

MO-11
DO-12
OF-13
OL-14
DL-15
ML-16
MF-17

MOD-21
MOL-22
MOF-23
DOL-24
DOF-25

MODL-31
MODF-32
MOFL-33
DOFL-34

MODFL-41

Pin-51
Gold-52
Out-53
Bridge-54

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A model of cusp breakdown in the eight functional molars and eight bicuspids of career officers in military service was developed on the basis of empirical data obtained from complete dental histories of career officer personnel. Using this model and cost estimates of materials and labor, it was determined that the expected cost of silver restorations will exceed the expected cost of a gold restoration at a minimum service life of twenty years with the expected cost of silver restorations becoming increasingly

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